Robust wavelet-based super-resolution reconstruction: Theory and Algorithm

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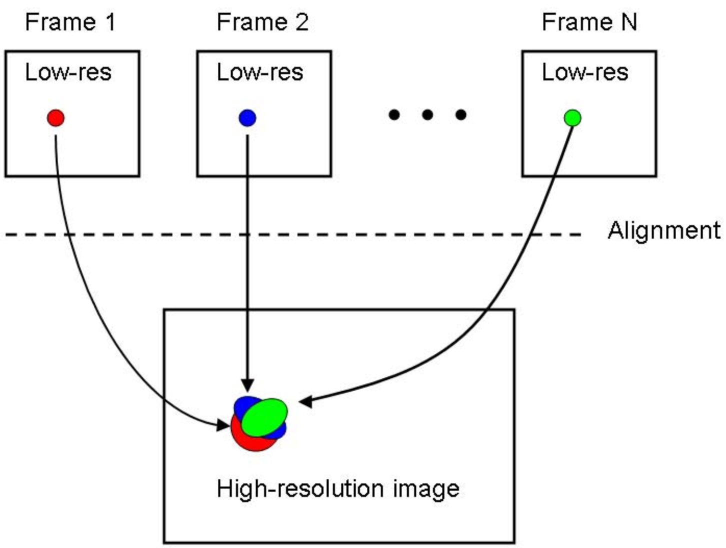
Super-resolution imaging

- Problem statement

How to reconstruct a high-resolution image from a sequence of low-resolution images



•Two key components

•Accurate image alignment

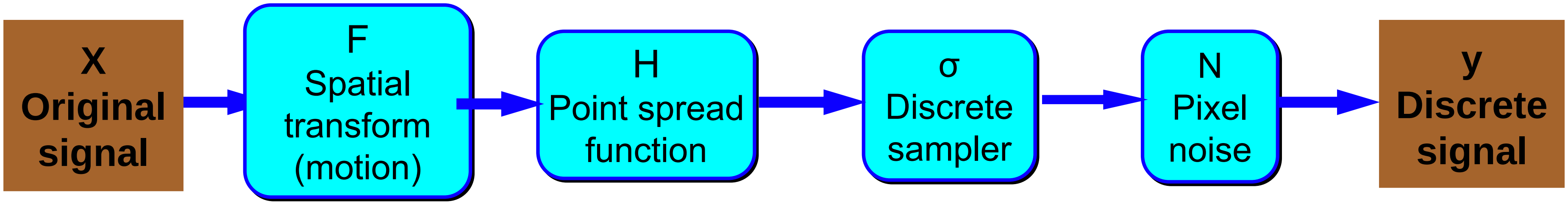
Homography-based image alignment with high accuracy

•Robust low-to-high resolution signal reconstruction scheme

Wavelet-based reconstruction with de-noising operator

Modeling image formation

$$y = \sigma[H * X(F(t))] + N$$



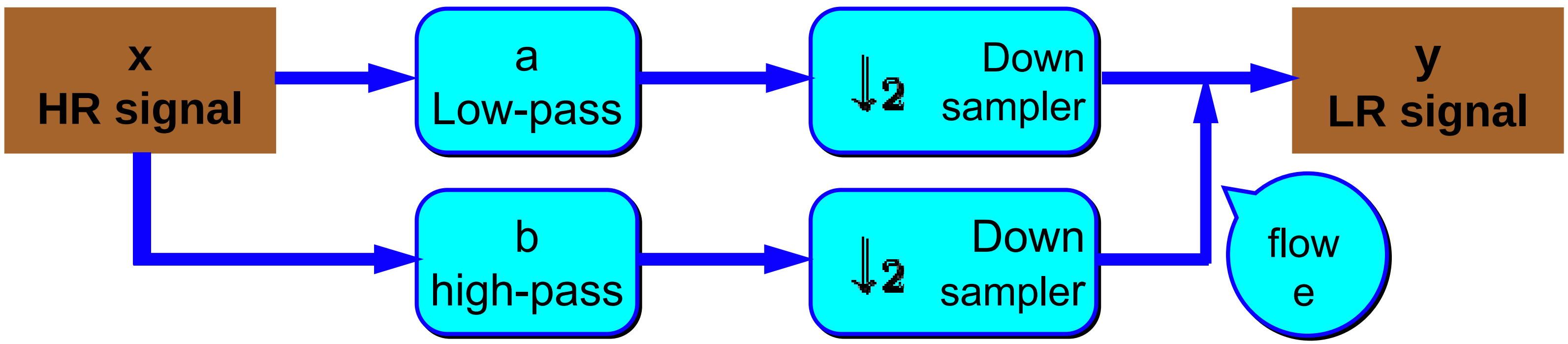
Modeling relationship between LR signal y and HR signal x

- New model of the relation between low and high resolution images using filter bank theory
  - 1D case:

Taylor expansion:

$$\begin{aligned} y &= [H * X(F(t))] \\ &= [H * X(\epsilon(t) + t)] \\ &= [H * X(t)] + \epsilon[H * X'(t)] \\ &= [H * X(t)] + \epsilon[H' * X(t)]. \end{aligned}$$

$$y = [a * x] \downarrow_2 + e \cdot [b * x] \downarrow_2$$



Limit on Super-resolution imaging

- Examples: Box-type PSF

$$a = \tfrac{1}{4}(1,2,1), \qquad a(z) = \tfrac{1}{4z^2}(1+z)^2$$
$$b = \tfrac{1}{4}(1,0,-1), \quad b(z) = \tfrac{1}{4z^2}(1-z^2)$$

- Observation: a(z) and b(z) have a common factor c(z)=(1+z)

- Limitation: At most, a blurred version of x can be recovered for any PSF

$$x^{hr} = c * x$$

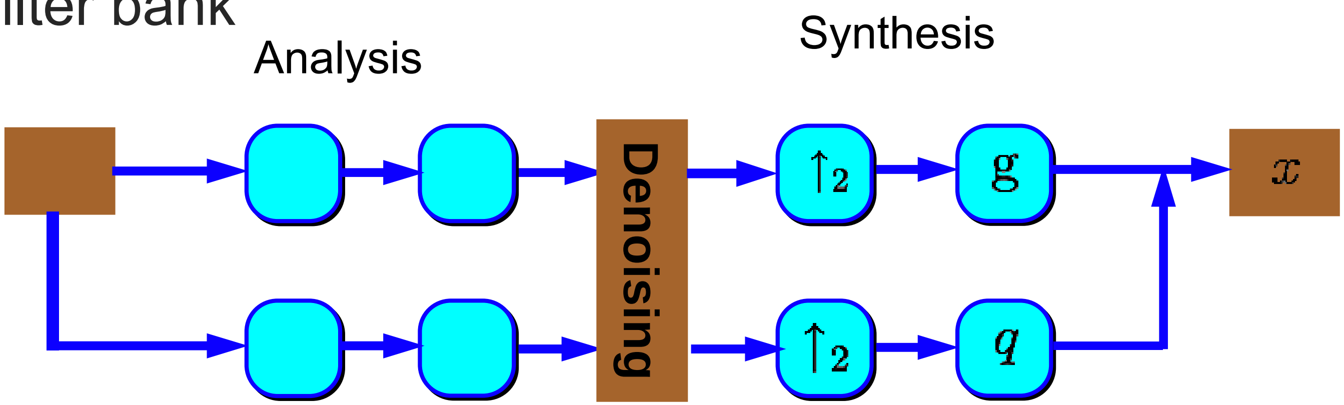
- 2D case for the example:

$$\mathrm{c}=\begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

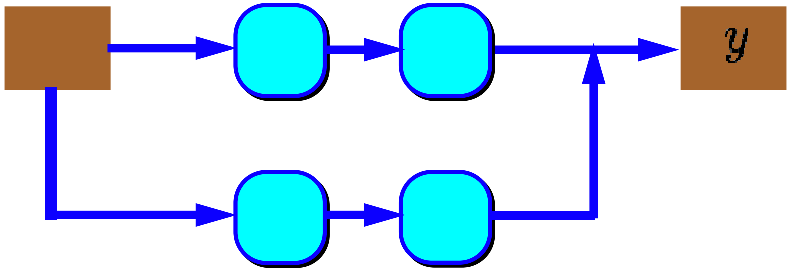
Basic idea

- Goal: reconstruct $x^{hr} = c * x$

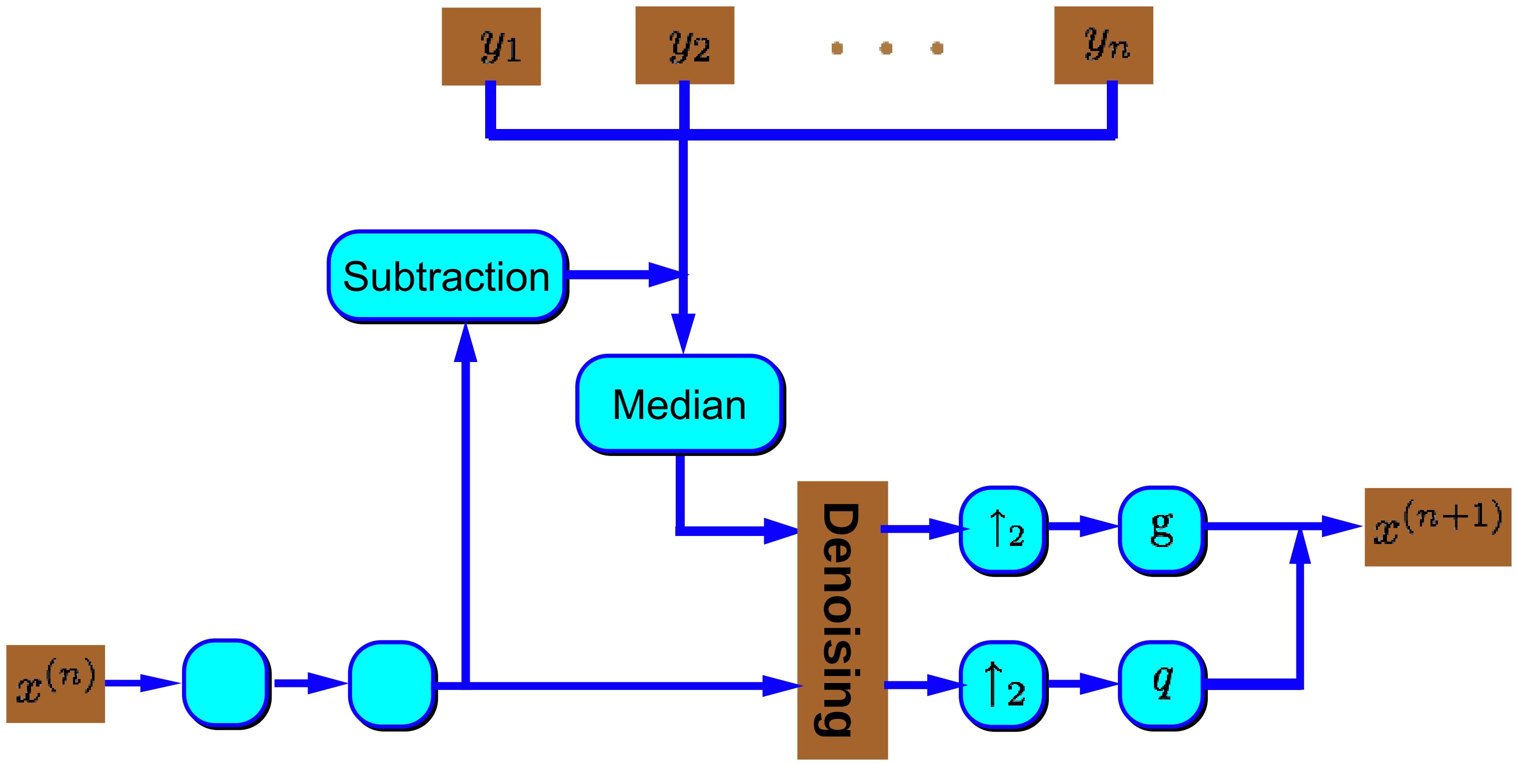
- Wavelet filter bank



- Recall the relationship between low and high resolution signal



Reconstruction scheme



- Hybrid shrinkage operator in Denoising
- Robust median regression

Flow estimation

Observation: the shape changes only with rotation

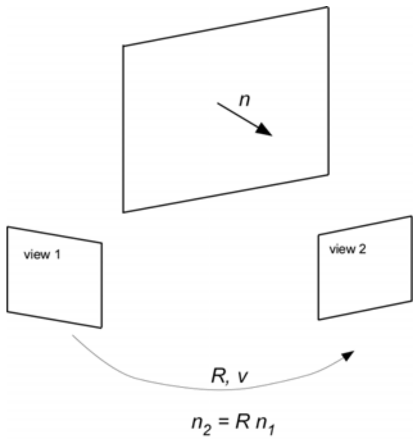


Image frames : $I_0, I_1, I_2, I_3, \ldots . I_k$

homography $P_k = R_k + \vec{v}_k \vec{n}^t$

Flow estimation from multiple frames

$$(\frac{dI}{d\vec{r}})^t \vec{u}(\vec{r}) = -\frac{dI}{dt}$$ Optic flow constraint

$$(\frac{dI}{d\vec{r}})^t (\vec{p}_k(\vec{r}) - \vec{p}_k^j(\vec{r})) = I_k(\vec{r}) - I_0(p_k^j(\vec{r}))$$

$$A_k(P_k^j)\vec{p}_k = 0$$

$$\min_{R_k,\vec{n},v_k} \sum_k \|A_k(P_k^j) vec[R_k + \vec{v}_k \vec{n}^t]\|^2$$

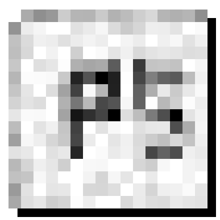
subject to the constraints that the R_ks are rotation matrices and $\|\vec{n}\| = 1$

Given P_k^j at step j, we compute $P_k^{j+1} = R^{j+1} + \vec{v}_k^{j+1}(\vec{n}^{j+1})^t$ at step $j+1$

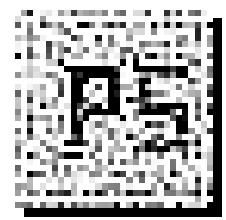
Experiments

- Synthesized data

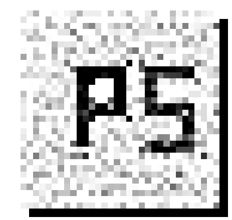
Original



Low resolution

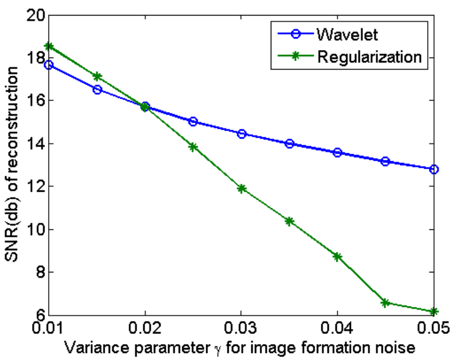


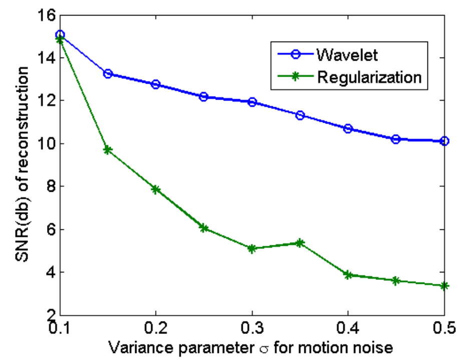
Tikhonov

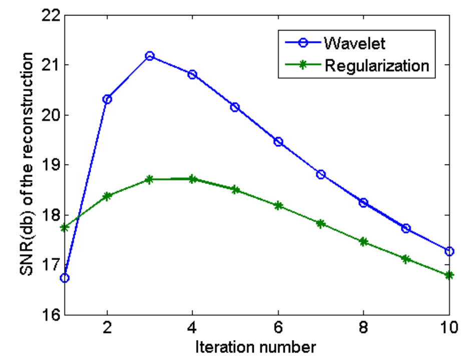


Wavelet

- Comparison between Tikhonov and wavelet-based regularization







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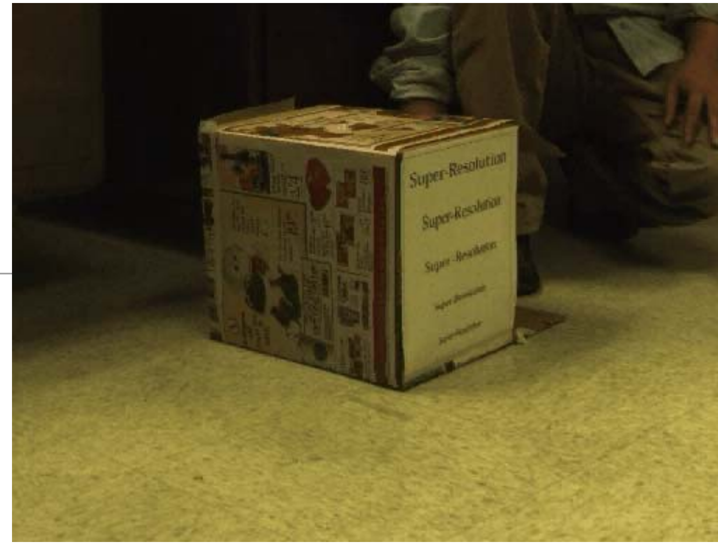
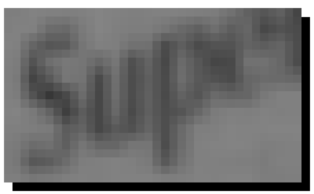
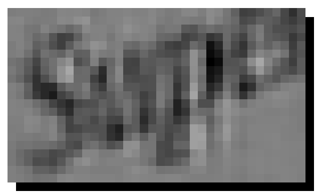
- Real video

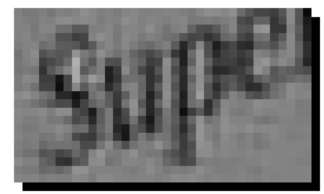




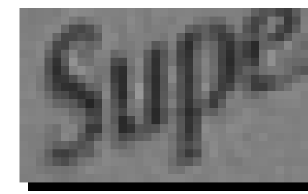
Interpolation



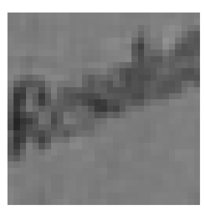
POCS+Affine

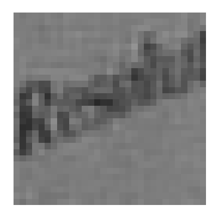


POCS+Homography



Wavelet+Homography





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Interpolation

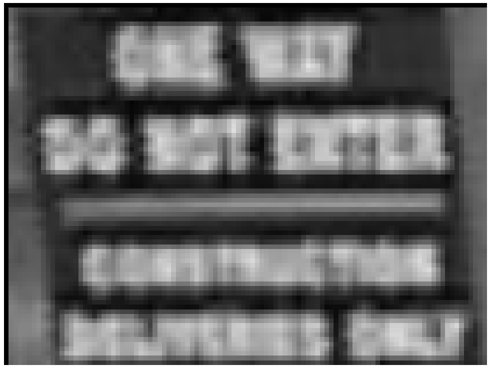
Irani's + Affine

Irani's + Homography

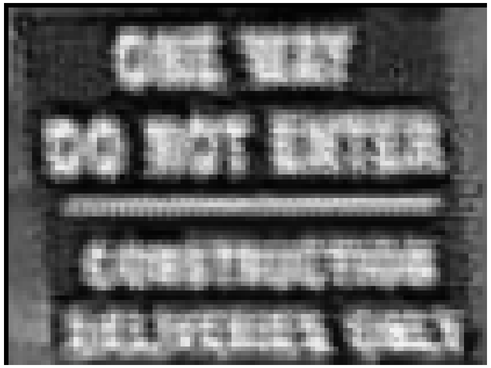
Wavelet + Homography

(cont')

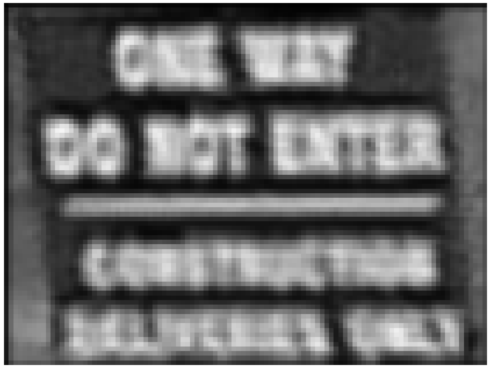
Key frame



Interpolation



Irani's + Affine



Wavelet + Affine